Q1 A.

Start with an empty stack of integers. You will attempt to do a sequence of pushes and pops so that the sequence of pops will be a specified permutation of 1, 2, 3 ,4 , 5, 6. You will be able to do exactly 6 push operations and 6 pop operations. The first push pushes 1 onto the stack; the next pushes 2; and so forth. The sixth push pushes 6 onto the stack.

For this exercise, we will let S denote a push operation and X a pop operation. Example:

The sequence SSSSSSXXXXXX outputs 654321.

1. Describe a sequence of pushes and pops that would produce output 325641 (or explain why it is not possible)

**Ans:**

The output sequence 325641 cannot be generated using the given push (S) and pop (X) operations due to the Last In, First Out (LIFO) nature of a stack. For 3 to be removed before 2 and 5, 6 would need to be pushed earlier, which conflicts with the required output order, making the sequence invalid.

1. Describe a sequence of pushes and pops that would produce output 154623 (or explain why it is not possible)

**Ans:**

| **Step** | **Operation** | **Stack (Top → Bottom)** | **Output** |
| --- | --- | --- | --- |
| 1 | Push 1(S) | 1 |  |
| 2 | Push 2(S) | 2 1 |  |
| 3 | Push 3(S) | 3 2 1 |  |
| 4 | Pop(X) | 2 1 | 3 |
| 5 | Push 4(S) | 4 2 1 |  |
| 6 | Push 5(S) | 5 4 2 1 |  |
| 7 | Pop(X) | 4 2 1 | 5 |
| 8 | Pop(X) | 2 1 | 4 |
| 9 | Push 6(S) | 6 2 1 |  |
| 10 | Pop(X) | 2 1 | 6 |
| 11 | Pop(X) | 1 | 2 |
| 12 | Pop(X) |  | 1 |

**Operation Sequence**: S S S X S S X X S X X X

**Final Output**: 1 5 4 6 2 3

Q1 B.

Suppose we store n keys in a hash table of size m = n^2 using a hash function h randomly chosen from a Universal class H of hash functions. Assume that X is a random variable that counts the number of collisions. Show that the Expected number of Collisions is < 1/2.

Given:

* + m=n2 (size of the hash table).
  + Universal hash function h.

Let X be the random variable representing collisions. The expected number of collisions is calculated as follows:

E[X]= n(n−1)/2m

Substituting m=n2:

E[X]=n(n−1)/2n2  = n−1/2n

For n≥1, E[X] < 1/2. Hence, the expected number of Ecollisions is less than 1/2.